

# On the possibility of blue tensor spectrum within single field inflation

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We present a series of theoretical constraints on the potentially viable inflation models that might yield a blue spectrum for primordial tensor perturbations. By performing a detailed dynamical analysis we show that, while there exists such possibility, the corresponding phase space is strongly bounded. Our result implies that, in order to achieve a blue tilt for inflationary tensor perturbations, one may either construct a non-canonical inflation model delicately, or study the generation of primordial tensor modes beyond the standard scenario of single slow-roll field.

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## I. INTRODUCTION

In recent years, the measurements of the cosmic microwave background (CMB) temperature anisotropies verified a nearly scale-invariant power spectrum of the primordial curvature perturbation to high precision [1]. This observational fact is highly consistent with the predictions from the perturbation theory of inflationary cosmology [2]. Therefore, inflation, which originally appeared in early 80's [3] (see also [4]), has become the most prevailing paradigm of describing the very early universe. Furthermore, inflationary cosmology also predicts a nearly scale-invariant power spectrum of the primordial gravitational waves, of which the magnitude is relatively smaller than that of the primordial curvature perturbation [5]. If these primordial tensor fluctuations exist, they could lead to the  $B$ -mode polarization signals in the CMB [6] and hence are expected to be observed in cosmological surveys.

So far there is no observational evidence that indicates existence of the primordial tensor fluctuations [7–9]. However, as was pointed out in [10], a suppression of power in the  $B$ -mode angular power spectrum at large scales might exist, which implies that a spectrum of primordial gravitational waves could have a blue tilt. Thus, from the perspective of theoretical interpretations, it is interesting to investigate whether a power spectrum of primordial gravitational waves with a blue tilt can be achieved in the framework of inflationary cosmology.

This question has already drawn the attention of cosmologists in the literature, and a couple of different mechanisms were put forward, namely the beyond-slow-roll inflation [11], the matter-bounce inflation [12], inflation with non-Bunch-Davis vacuum [13], the non-commutative field inflation [14], the variable gravity quintessential inflation [15], the string gas cosmology [16], or the Hawking radiation during inflation [17]. Therefore, a careful characterization of the power spectrum of the primordial  $B$ -mode polarization is very important to falsify the paradigms of very early universe (see [18] for the characterization of the primordial gravitational waves within various very early universe models).

In the present work we make a remark on the potential challenge of regular inflation models to generate a blue tilt for the primordial gravitational waves. We restrict ourselves within the standard general relativity and present a potential resolution to this challenge by proposing to extend the parameter space of inflation models by including

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non-canonical operators. In particular, we phenomenologically consider a class of inflation models with the Horndeski operator being involved. Such models were considered in inflationary cosmology for the purpose of circumventing the paradigm of Higgs inflation [19], and are dubbed as “G-inflation” [20] (see for example [21] for generalized analyses and see [22] for a counter-claim from the stability viewpoint). In our construction, differing from the application of the Galilean symmetry, inflation is driven by a scalar field with a Horndeski operator which could be achieved either by the kinetic term or the potential energy. We investigate the dynamics of this cosmological system by performing a detailed phase space analysis. We find that in general the generation of a blue tilt of the primordial gravitational waves in a viable inflation model is difficult since the expected trajectories are not stable in the phase space. However, a short period of super-inflationary phase might be possible and thus would circumvent the above theoretical challenges.

The article is organized as follows. In Section II, we briefly review the standard picture of predictions made by regular inflation models on the primordial curvature and tensor perturbations. We can see that it is forbidden to produce a power spectrum of the primordial tensor modes with a blue tilt in a wide class of inflation models. Then, in Section III we present a class of extended inflation models by including a parameterized Horndeski operator. By selecting several typical inflation potentials, we perform the dynamical analyses in details and derive their attractor solutions. We show that the inflationary trajectories with a blue tilt do not correspond to these stable solutions. We conclude with a discussion in Section IV. Throughout the article we take the sign of the metric  $(+, -, -, -)$  and define the reduced Planck mass by  $m_{\text{Pl}} = 1/\sqrt{8\pi G}$ .

## II. GENERAL DISCUSSIONS

In the paradigm of inflationary cosmology, both the primordial curvature perturbation and gravitational waves are originated from quantum fluctuations in a nearly exponentially expanding universe at high energy scales. During this expansion, the physical wavelengths of the metric fluctuations are stretched out of the Hubble radius and then form power spectra as observed in the CMB. Such a convenient causal mechanism of generating primordial perturbations can determine their power spectra by a series of simple relations.

Particularly, for a general inflation model with a k-essence Lagrangian [23], the power spectrum of the curvature perturbation  $\mathcal{R}$  is determined by four parameters, namely the Hubble rate  $H$ , the spectral index  $n_{\mathcal{R}}$ , the slow-roll parameter  $\epsilon$ , and the sound speed parameter  $c_s$ , through the following relation

$$\mathcal{P}_{\mathcal{R}} = \frac{\xi_H}{8\pi^2 \epsilon c_s} \left( \frac{k}{k_0} \right)^{n_{\mathcal{R}} - 1}, \quad (1)$$

with

$$\xi_H \equiv \frac{H_I^2}{m_{\text{Pl}}^2}, \quad (2)$$

where  $k_0$  is the pivot scale. The subscript  $I$  denotes that the value of the Hubble parameter is taken during the inflationary stage. The slow roll parameter  $\epsilon$  is defined by

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad (3)$$

where dots denote derivatives with respect to  $t$ , and thus it is determined by the background dynamics of inflation. The sound speed parameter  $c_s$  characterizes the propagation of primordial scalar fluctuations. Theoretically, its value is constrained between 0 and 1 so that the model is free from the gradient instability and super-luminal propagation (see however [24] for a different viewpoint on super-luminal propagation of a k-essence field). Moreover, the recent non-detection of the primordial non-Gaussianity by the Planck data [25] implies that  $c_s$  cannot be too small. The spectral index  $n_{\mathcal{R}}$  can be derived straightforwardly from its definition through

$$n_{\mathcal{R}} - 1 \equiv \frac{d \log \mathcal{P}_{\mathcal{R}}}{d \log k} = -4\epsilon + 2\eta - s, \quad (4)$$

where we have introduced two more slow-roll parameters, namely

$$\eta \equiv \epsilon - \frac{\dot{\epsilon}}{2H\epsilon}, \quad s \equiv \frac{\dot{c}_s}{Hc_s}. \quad (5)$$

According to the current CMB observations, the spectral index  $n_{\mathcal{R}}$  takes a value which is slightly less than unity and hence the power spectrum of the primordial curvature perturbation is red-tilted.

For the primordial tensor fluctuations, the associated relations are even simpler if the gravity theory is still general relativity. The corresponding power spectrum takes the form of

$$\mathcal{P}_T = \frac{2}{\pi^2} \xi_H \left( \frac{k}{k_0} \right)^{n_T}, \quad (6)$$

and therefore it is easy to see that the amplitude of the primordial tensor fluctuations only depends on the inflationary Hubble parameter and the corresponding spectral index  $n_T$ , where the latter, by definition, is given by

$$n_T \equiv \frac{d \log \mathcal{P}_T}{d \log k} = -2\epsilon. \quad (7)$$

Expressions (6) and (7) are generic to any single field inflation model minimally coupled to gravity. Since the Hubble rate  $H$  is monotonically decreasing in regular inflation models ( $\dot{H} < 0$ ) it is implied that  $\epsilon > 0$ , and hence one can conclude that the spectral index of the primordial tensor fluctuations  $n_T$  in these models is always negative. Therefore it is red-tilted too, as it is the case of the curvature perturbation.

Although the present observations cannot determine the tilt of  $n_T$ , we shall notice that if one expects a slightly blue power spectrum for the inflationary tensor fluctuations,  $\epsilon$  has to be efficiently negative. This phenomenon implies a violation of weak/null energy condition during inflation. In the literature there have been some proposals to give rise to the corresponding energy condition violation in very early universe, namely super-inflation by the nonlocal gravity approach [26], super-inflation in loop quantum cosmology [27], inflation in (super-)renormalizable gravity [28], as well as from a general viewpoint of effective field approach [29].

However, it is not trivial to achieve an inflationary model that can realize  $\epsilon > 0$  in a stable way, without any pathologies, in the framework of Einstein gravity<sup>1</sup>. In particular, one ought to be aware of the following theoretical constraints:

- First of all, the model must be stable against any ghost mode, in order for the perturbation theory describing the primordial perturbations generated from vacuum fluctuations to be reliable.
- The curvature perturbation must be free of the gradient instability, or at least experience this instability within a very short period. In this regard, there is no harmful growth of the primordial perturbations that might violate the current observational constraints.
- The spacetime symmetry of the universe should recover the Lorentz symmetry. This indicates that the theory of matter fields has to recover the canonical version, with all higher-order operators being suppressed at low energy scales.
- After inflation, the universe needs to gracefully exit to the regular thermal expanding phase smoothly. Hence it is implied that the weak energy condition has to be recovered at late times of the inflationary stage or after.

Keeping these theoretical requirements in mind, it is interesting to look for a viable inflation model that generates a power spectrum of the primordial tensor fluctuations with a blue tilt and is consistent with latest cosmological observations. This is exactly the goal of the present work.

### III. INFLATION WITH A HORNDESKI OPERATOR

The violation of the weak energy condition in a stable way, is of theoretical interest in various models of very early universe physics. One plausible mechanism of achieving such a scenario is to make use of a ghost condensate field, in which the kinetic term for the inflaton takes a non-vanishing expectation value in the infrared regime [32]. However, this type of models often suffers from a gradient instability, when the universe exits from the inflationary phase to the normal thermal expansion. Another approach to realize the weak energy condition violation is to make use of a Galileon-type field (also dubbed as the Horndeski field) [33]. The key feature of this type of field is that it contains higher-order derivative terms in the Lagrangian, while the equations of motion remain second order and thus do not necessarily lead to the appearance of ghost modes. These important features have led to many recent studies of Galileon models which yield a period of inflationary phase at early times of the universe [20–22, 34].

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<sup>1</sup> It is known that any single field described by a k-essence type Lagrangian cannot break the null energy condition without pathologies: see e.g. the appendix of [30] as well as a comprehensive review [31].

In this section, we focus on a class of inflation model with a Horndeski operator. We phenomenologically consider a dimensionless scalar field  $\phi$  with a Lagrangian of the type

$$\mathcal{L} = \frac{m_{\text{Pl}}^2}{2} R + K(\phi, X) + G(\phi, X) \square \phi, \quad (8)$$

in which  $K$  is a k-essence type operator and  $G$  is a Horndeski operator. They both are functions of  $\phi$  and the kinetic term

$$X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi, \quad (9)$$

and  $\square \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu$  is the standard d'Alembertian operator. This type of Lagrangian, with specifically chosen forms of  $K$  and  $G$ , was adopted to drive the late-time acceleration of the universe in [35], and its dynamical analysis was carried out in [36]. Additionally, in [37] it was found that if one combines the ghost condensate and Horndeski operators he can obtain a healthy bouncing cosmology, with a smooth transition from a contracting universe to standard expanding radiation and matter dominated phases (see also [38] for extended studies).

To be specific, we choose the following *minimal* ansatz:

$$K(\phi, X) = m_{\text{Pl}}^2 X - V(\phi), \quad (10)$$

$$G(\phi, X) = m_{\text{Pl}}^2 \gamma(\phi) \left( \frac{2X}{m_{\text{Pl}}^2} \right)^p, \quad (11)$$

where  $\gamma(\phi)$  is a dimensionless function of the inflaton field and  $p$  is a coefficient as a free model parameter. Note that the expression of  $K$  in (10) corresponds to the canonical Lagrangian for the inflaton field. Moreover, the Horndeski operator  $G$  is expected to stabilize the propagation of the curvature perturbation when the inflationary stage with  $\epsilon < 0$  occurs. Its effect is automatically suppressed at low energy scales if we choose  $p$  to be positive definite or  $\gamma(\phi)$  to decay rapidly. Consequently, the model under consideration could partly satisfy the theoretical limits pointed out at the end of the previous section.

### A. Background equations of motion

Varying the Lagrangian with respect to the metric, and focusing on a flat Friedmann-Robertson-Walker (FRW) geometry of the form  $ds^2 = dt^2 - a^2(t) \delta_{ij} dx^i dx^j$ , with  $a(t)$  the scale factor, leads to the Friedmann equations

$$H^2 = \frac{\rho_\phi}{3m_{\text{Pl}}^2}, \quad \dot{H} = -\frac{\rho_\phi + P_\phi}{2m_{\text{Pl}}^2}, \quad (12)$$

where the energy density and the pressure of the scalar field respectively are written as

$$\rho_\phi = \frac{1}{2} m_{\text{Pl}}^2 \dot{\phi}^2 \left[ 1 + \frac{12p\gamma H \dot{\phi}}{m_{\text{Pl}}^2} \left( \frac{\dot{\phi}^2}{m_{\text{Pl}}^2} \right)^{p-1} - 2\gamma_\phi \left( \frac{\dot{\phi}^2}{m_{\text{Pl}}^2} \right)^p \right] + V(\phi), \quad (13)$$

$$P_\phi = \frac{1}{2} m_{\text{Pl}}^2 \dot{\phi}^2 \left[ 1 - \frac{4p\gamma \ddot{\phi}}{m_{\text{Pl}}^2} \left( \frac{\dot{\phi}^2}{m_{\text{Pl}}^2} \right)^{p-1} - 2\gamma_\phi \left( \frac{\dot{\phi}^2}{m_{\text{Pl}}^2} \right)^p \right] - V(\phi). \quad (14)$$

Moreover, the equation of motion for the scalar field can be derived as

$$\mathcal{P} \ddot{\phi} + \mathcal{D} \dot{\phi} + V_\phi = 0, \quad (15)$$

where we have introduced

$$\mathcal{P} = m_{\text{Pl}}^2 \left[ 1 + \frac{12p^2 \gamma H \dot{\phi}}{m_{\text{Pl}}^2} \left( \frac{\dot{\phi}^2}{m_{\text{Pl}}^2} \right)^{p-1} - 2(1+p)\gamma_\phi \left( \frac{\dot{\phi}^2}{m_{\text{Pl}}^2} \right)^p + 6p^2 \gamma^2 \left( \frac{\dot{\phi}^2}{m_{\text{Pl}}^2} \right)^{2p} \right], \quad (16)$$

$$\mathcal{D} = 3Hm_{\text{Pl}}^2 \left\{ 1 + \frac{6\gamma H \dot{\phi}}{m_{\text{Pl}}^2} \left( \frac{\dot{\phi}^2}{m_{\text{Pl}}^2} \right)^{p-1} + \left[ 2(p-1)\gamma_\phi - p\gamma \frac{\dot{\phi}}{H} - \gamma_{\phi\phi} \frac{\dot{\phi}}{3H} \right] \left( \frac{\dot{\phi}^2}{m_{\text{Pl}}^2} \right)^p + \left( 2\gamma\gamma_\phi \frac{\dot{\phi}}{H} - 6p^2 \gamma^2 \right) \left( \frac{\dot{\phi}^2}{m_{\text{Pl}}^2} \right)^{2p} \right\}. \quad (17)$$

Note that, the positivity of the coefficient  $\mathcal{P}$  can be applied to examine whether the model suffers from a ghost or not. On the other hand, the coefficient  $\mathcal{D}$  is an effective friction term for the inflaton field. It is easy to check that the regular Klein-Gordon equation in a FRW background can be recovered if one takes  $\gamma = 0$ . Finally, for completeness, in Appendix A we provide the cosmological equations for general  $K(\phi, X)$  and  $G(\phi, X)$ .

In order to analyze the background dynamics it proves convenient to introduce various rolling parameters as:

$$\epsilon_\phi \equiv \frac{\dot{\phi}^2}{2H^2}, \quad \eta_\phi \equiv -\frac{\ddot{\phi}}{H\dot{\phi}}, \quad \xi_\gamma \equiv \frac{\dot{\gamma}}{H\gamma}, \quad \eta_\gamma \equiv \frac{\dot{\xi}_\gamma}{H\xi_\gamma}, \quad (18)$$

which are all dimensionless. Note that the first two parameters  $\epsilon_\phi$  and  $\eta_\phi$  are mainly associated directly with the dynamics of the inflaton field. In traditional inflation models with a canonical kinetic term, they coincide with the regular slow-roll parameters  $\epsilon$  and  $\eta$  as provided in (3) and (5). The last two parameters of (18)  $\xi_\gamma$  and  $\eta_\gamma$  respectively describe the first and second order variation of the coefficient  $\gamma$  within each Hubble time. In summary, using these parameters one can rewrite the energy density and the pressure of the scalar field as

$$\begin{aligned} \rho_\phi &= V(\phi) + m_{\text{Pl}}^2 H^2 \left[ \epsilon_\phi + \gamma(6p - \xi_\gamma)(2\epsilon_\phi)^{p+1/2} \xi_H^p \right], \\ P_\phi &= -V(\phi) + m_{\text{Pl}}^2 H^2 \left[ \epsilon_\phi + \gamma(2p\eta_\phi - \xi_\gamma)(2\epsilon_\phi)^{p+1/2} \xi_H^p \right], \end{aligned} \quad (19)$$

where  $\xi_H$  has been introduced in (2).

Recalling that the definition of the background slow-roll parameter  $\epsilon$  (3), and inserting (19) into the Friedmann equations (12), we can extract the relation between  $\epsilon$  and  $\epsilon_\phi$  as

$$\epsilon = \epsilon_\phi \left[ 1 + 2\gamma(3p - \xi_\gamma + p\eta_\phi)(2\epsilon_\phi)^{p-1/2} \xi_H^p \right]. \quad (20)$$

Note that when  $\gamma$  vanishes  $\epsilon$  is equal to  $\epsilon_\phi$  and hence we reduce to the canonical case where  $\epsilon > 0$ . However, in the general case the dynamics of  $\gamma$  can realize  $\epsilon < 0$ , despite the fact that  $\epsilon_\phi > 0$ . Hence, the above model can indeed give rise to a blue tilt, as discussed in the previous section. In the rest of this work we study such a possibility.

## B. Cosmological perturbations and ghost avoidance

A general feature of cosmological scenarios which involve higher-order derivatives is that they can exhibit ghost instabilities at perturbation level, which can be treated in the context of an appropriate field redefinition [39]. Hence, before using (8) for the description of the inflationary phase, one needs to perform a detail perturbation analysis and extract the necessary conditions for the avoidance of ghosts and gradient instabilities. Following [40] and applying the ansatzes of (10) and (11), we deduce that in a FRW background the condition for ghost absence writes as

$$Q_s \equiv \frac{w_1(4w_1w_3 + 9w_2^2)}{3w_2^2} \geq 0, \quad (21)$$

where

$$\begin{aligned} w_1 &= w_4 = m_{\text{Pl}}^2, \\ w_2 &= 2G_X X \dot{\phi} + 2m_{\text{Pl}}^2 H = 2m_{\text{Pl}}^2 \left[ H + p\gamma \dot{\phi} \left( \frac{\dot{\phi}^2}{m_{\text{Pl}}^2} \right)^p \right], \\ w_3 &= -9m_{\text{Pl}}^2 H^2 + 3X(K_X + 2XK_{XX}) + 6X \left( G_\phi + XG_{\phi X} - 6H\dot{\phi}G_X - 3XH\dot{\phi}G_{XX} \right) \\ &= \frac{3}{2}m_{\text{Pl}}^2 \dot{\phi}^2 \left[ 1 + (p+1) \left( \frac{\dot{\phi}^2}{m_{\text{Pl}}^2} \right)^p \left( \gamma_\phi - 6p\gamma \frac{H}{\dot{\phi}} \right) \right] - 9m_{\text{Pl}}^2 H^2. \end{aligned} \quad (22)$$

Note that the physical meaning of the  $Q_s$  parameter corresponds to the positivity coefficient of the perturbation variable which appears in Eq. (B2) in Appendix B.

In addition, the condition for the avoidance of gradient instabilities (associated with the scalar field propagation speed) reads

$$c_s^2 \equiv \frac{3(2w_1^2w_2H - w_2^2w_4 + 4w_1w_2\dot{w}_1 - 2w_1^2\dot{w}_2)}{w_1(4w_1w_3 + 9w_2^2)} \geq 0. \quad (23)$$

Note that the above two theoretical constraints can impose the bound of the parameters for inflation models in the literature directly. To be explicit, the condition for ghost absence (21) requires that

$$w_3 \geq -\frac{9w_2^2}{4w_1},$$

and the condition for the avoidance of gradient instabilities (23) implies

$$0 \leq w_2 \leq 2Hw_1,$$

under the assumption of  $|\dot{w}_2/w_2H| \ll 1$ . As a result, the parameter space of inflation models that attempt to generate a blue spectrum of primordial gravitational waves could be strongly constrained by the above two theoretical requirements.

Furthermore, we would like to provide several examples of explicit inflation models, in order to demonstrate how these theoretical conditions can constrain the validity of these models. For instance, for a model of canonical inflation with  $\gamma = 0$ , these two conditions are satisfied automatically, since  $w_2 = 2Hw_1$  exactly, however the spectrum of tensor modes is also red tilted; moreover, if we consider an explicit model with  $p = 1$  and  $\gamma = \gamma_0 e^{\lambda_\gamma \phi}$  and take a positive value for  $\dot{\phi}$ , then the theoretical constraints yield approximately that  $\gamma_0 \leq 0$  and  $2\gamma_0 \lambda_\gamma e^{\lambda_\gamma \phi} \dot{\phi}^2 + m_{\text{Pl}}^2 > 0$ . For the latter case, there might exist possible parameter space that allows for a blue tilt for tensor modes, however, as will be shown in the following subsection, this possibility is unstable under the phase space analysis.

We refer to Appendix B for a specific instruction of the perturbation analysis for the inflation model under consideration.

### C. Dynamical analysis

Let us now apply the powerful method of dynamical analysis [41–43] in order to investigate the general features of inflation in the scenario at hand. In order to perform such a stability analysis we first transform the cosmological equations in their autonomous form  $\mathbf{X}' = \mathbf{f}(\mathbf{X})$ , where primes denote derivative with respect to  $\log a$ , with  $\mathbf{X}$  a vector constituted by suitable auxiliary variables and  $\mathbf{f}(\mathbf{X})$  the corresponding vector of the autonomous equations. The critical points  $\mathbf{X}_c$  of this autonomous system are extracted through the condition  $\mathbf{X}' = \mathbf{0}$ . Their stability is examined by expansion around them as  $\mathbf{X} = \mathbf{X}_c + \mathbf{V}$ , with  $\mathbf{V}$  the vector of the variable perturbations, resulting to the perturbation equations of the form  $\mathbf{V}' = \mathbf{Q} \cdot \mathbf{V}$ , with the matrix  $\mathbf{Q}$  containing all the coefficients of these equations. Therefore, the type and properties of a specific critical point are determined by the eigenvalues of  $\mathbf{Q}$ : eigenvalues with positive real parts imply instability, eigenvalues with negative real parts imply stability, while eigenvalues with real parts of different sign correspond to a saddle point. In this way, one is able to extract qualitative information for the global dynamics of the examined scenario, independently of the initial conditions and the specific universe evolution.

We are interesting in analyzing the Friedmann equations (12), along with the scalar field evolution equation (15). Considering for simplicity the most important case where  $p = 1$ , we introduce the auxiliary variables

$$x \equiv \frac{\dot{\phi}}{\sqrt{6}H}, \quad y \equiv \frac{\sqrt{V(\phi)}}{\sqrt{3}m_{\text{Pl}}H}, \quad z \equiv \frac{\gamma(\phi)H\dot{\phi}}{m_{\text{Pl}}^2}. \quad (24)$$

In terms of these variables the first Friedmann equation takes the form

$$(1 + 12z)x^2 + y^2 - 2\sqrt{6}zx^3 \frac{\gamma_\phi}{\gamma} = 1, \quad (25)$$

while from the definitions of  $x$  and  $z$  we acquire  $H^2 = m_{\text{Pl}}^2 z / (\sqrt{6}x\gamma)$ .

In order to proceed the analysis we have to consider ansatzes for the potential  $V(\phi)$  and the Galileon coupling function  $\gamma(\phi)$ . As a simple model we analyze the exponential potential

$$V(\phi) = V_0 e^{\lambda_V \phi}, \quad (26)$$

which is widely used in the literature [42, 44], along with an exponential coupling function

$$\gamma(\phi) = \gamma_0 e^{\lambda_\gamma \phi}. \quad (27)$$

We comment that the specific forms considered above are very representative and can grasp common features obtained in other cases too. Firstly, the exponential potential is one of the most representative ansatzes used in cosmology. In particular, this exponential potential possesses a manifest advantage that it can mimic any arbitrary cosmic evolution

with a constant equation of state. Additionally, it is well known that an exponential potential can yield an attractor solution. Secondly, from the discussion performed below Eq. (23), one can see that one important theoretical bound for  $\gamma$  is its positivity. This can be exactly implemented by taking the form of  $\gamma$  to be exponential, and accordingly, its positivity is only determined by the  $\gamma_0$  parameter without any sign change. As will be shown in the following dynamical analysis, the evolution of the inflaton field under this specific form can allow for fixed points in the trajectories of the inflaton field.

In this case, (12) and (15) are transformed to the autonomous form:

$$\begin{aligned} x' = & \left\{ 2 \left[ 1 + 4z \left( 3 + 9x^2z - \sqrt{6}x\lambda_\gamma \right) \right] \left[ -x^2 + 2zx^2 \left( \sqrt{6}x\lambda_\gamma - 6 \right) - y^2 \right] \right\}^{-1} \\ & \times \left\{ 6xy^2(1+6z) + \sqrt{6}\lambda_V y^4 + \sqrt{6}\lambda_V x^2 y^2(1+6z) + 24\sqrt{6}\lambda_\gamma x^4 z [1+3(3+y^2)z] \right. \\ & + 24\sqrt{6}\lambda_\gamma x^6 z^2 (6+54z+\lambda_\gamma^2) - 12x^3 z [3+18z+y^2(3+18z+\lambda_\gamma\lambda_V+\lambda_\gamma^2)] \\ & \left. - 12x^5 z [3+\lambda_\gamma^2+6z(9+36z+5\lambda_\gamma^2)] - 864\lambda_\gamma^2 x^7 z^3 \right\}, \end{aligned} \quad (28)$$

$$\begin{aligned} y' = & m_{\text{Pl}}xy \left\{ 2 \left[ 1 + 4z \left( 3 + 9x^2z - \sqrt{6}x\lambda_\gamma \right) \right] \left[ -x^2 + 2zx^2 \left( \sqrt{6}x\lambda_\gamma - 6 \right) - y^2 \right] \right\}^{-1} \\ & \times \left\{ 432\lambda_V\lambda_\gamma x^5 z^3 - \sqrt{6}\lambda_V y^2(1+18z) + 36x^3 z\lambda_\gamma(\lambda_V+12z\lambda_V-6z\lambda_\gamma) \right. \\ & - \sqrt{6}x^2 \{ [1+12z(2+12z+3zy^2)]\lambda_V - 36\lambda_\gamma z(1+8z) \} \\ & \left. - 12\sqrt{6}x^4 z^2\lambda_V(3+36z+4\lambda_\gamma^2) - 6x[1+4z(6+27z-y^2\lambda_V\lambda_\gamma)] \right\}, \end{aligned} \quad (29)$$

$$\begin{aligned} z' = & z \left\{ 2x \left[ 1 + 4z(3+9x^2z - \sqrt{6}x\lambda_\gamma) \right] \right\}^{-1} \\ & \times \left\{ 6x \left\{ -1 - 6z - x \left[ x + 36xz^2 \left( 2 - \sqrt{6}\lambda_\gamma x + \lambda_\gamma^2 x^2 \right) \right. \right. \right. \\ & \left. \left. \left. + z \left( \sqrt{6}y^2\lambda_V + 18x - 6\sqrt{6}\lambda_\gamma x^2 - 2\lambda_\gamma^2 x \right) \right] \right\} - \sqrt{6}\lambda_V y^2 \right\}. \end{aligned} \quad (30)$$

Additionally, in terms of the auxiliary variables, the energy density and pressure of the scalar field can be rewritten as follows:

$$\rho_\phi = 3m_{\text{Pl}}^2 H^2 \left\{ y^2 + x^2 \left[ 1 - 2z \left( \sqrt{6}x\lambda_\gamma - 6 \right) \right] \right\}, \quad (31)$$

$$\begin{aligned} P_\phi = & \frac{3m_{\text{Pl}}^2 H^2}{4z(9x^2z - \sqrt{6}\lambda_\gamma x + 3) + 1} \\ & \times \left\{ 24\lambda_\gamma^2 x^4 z^2 - 6\sqrt{6}(1+4z)z\lambda_\gamma x^3 + [12z(3z+2)+1]x^2 + 2\sqrt{6}y^2 xz(2\lambda_\gamma + \lambda_V) - y^2(12z+1) \right\}, \end{aligned} \quad (32)$$

and hence the equation of state of the scalar field takes the form

$$w_\phi = \frac{24\lambda_\gamma^2 x^4 z^2 - 6\sqrt{6}(1+4z)z\lambda_\gamma x^3 + [12z(3z+2)+1]x^2 - y^2(12z+1) + 2\sqrt{6}y^2 xz(2\lambda_\gamma + \lambda_V)}{\{y^2 + x^2 [1 - 2z(\sqrt{6}x\lambda_\gamma - 6)]\} [4z(9x^2z - \sqrt{6}\lambda_\gamma x + 3) + 1]}. \quad (33)$$

Note that knowing  $w_\phi$  we can straightforwardly calculate the deceleration parameter as  $q \equiv -1 - \dot{H}/H^2 = 1/2 + 3w_\phi/2$ . Finally, we can express the two instability-related quantities  $Q_s$  and  $c_s^2$  given in (21) and (23) in terms of the auxiliary variables as

$$Q_s = \frac{3x^2 [4z(9x^2z - \sqrt{6}\lambda_\gamma x + 3) + 1]}{(6xz^2 - 1)^2}, \quad (34)$$

$$\begin{aligned} c_s^2 = & \left\{ x \left[ 4z \left( 9x^2z - \sqrt{6}\lambda_\gamma x + 3 \right) + 1 \right]^2 \right\}^{-1} \left\{ 12z^2 x^3 (2\lambda_\gamma^2 + 12z + 5) - 24\sqrt{6}z^3 \lambda_\gamma x^4 \right. \\ & \left. - 432z^4 x^5 - 4\sqrt{6}(1+8z)z\lambda_\gamma x^2 + x + 4xz [3z(5-3y^2) + 2] - 2\sqrt{6}y^2 z\lambda_V \right\}. \end{aligned} \quad (35)$$

The real and physically meaningful critical points (namely those that correspond to an expanding universe, i.e. with  $H > 0$ ) of the autonomous system (28) - (30) are obtained by setting the left hand sides of these equations to zero, and are presented in Table I along with their existence conditions. For each of these critical points we calculate the

Points	$x_c$	$y_c$	$z_c$	Exist for	Eigenvalues	Stability
$A$	$+1$	$0$	$0$	always	$3, \sqrt{6}\lambda_\gamma - 6, 3 + \sqrt{\frac{3}{2}}\lambda_V$	unstable for $\lambda_V > -\sqrt{6}$ , $\lambda_\gamma > \sqrt{6}$ saddle point otherwise
$B$	$-1$	$0$	$0$	always	$3, -\sqrt{6}\lambda_\gamma - 6, 3 - \sqrt{\frac{3}{2}}\lambda_V$	unstable for $\lambda_V < \sqrt{6}$ , $\lambda_\gamma < -\sqrt{6}$ saddle point otherwise
$C$	$\frac{\lambda_\gamma + \sqrt{\lambda_\gamma^2 - 6}}{\sqrt{6}}$	$0$	$\frac{\alpha^- - 3}{18}$	$\lambda_\gamma^2 \geq 6$	$C^-, C^+, \frac{(\lambda_\gamma + \lambda_V)\alpha^+}{2\lambda_g}$	unstable for $\lambda_\gamma > \sqrt{6}$ , $\lambda_V > -\lambda_\gamma$ saddle point otherwise
$D$	$\frac{\lambda_\gamma - \sqrt{\lambda_\gamma^2 - 6}}{\sqrt{6}}$	$0$	$\frac{\alpha^+ - 3}{18}$	$\lambda_\gamma^2 \geq 6$	$D^-, D^+, \frac{(\lambda_\gamma + \lambda_V)\alpha^-}{2\lambda_g}$	unstable for $\lambda_\gamma < -\sqrt{6}$ , $\lambda_V < -\lambda_\gamma$ saddle point otherwise
$E$	$-\frac{\lambda_V}{\sqrt{6}}$	$\sqrt{1 - \frac{\lambda_V^2}{6}}$	$0$	$0 < \lambda_V^2 \leq 6$	$\lambda_V^2 - 3, -\lambda_V(\lambda_\gamma + \lambda_V), \frac{\lambda_V^2 - 6}{2}$	stable node for $-\sqrt{3} < \lambda_V < 0$ , $\lambda_\gamma < -\lambda_V$ stable node for $0 < \lambda_V < \sqrt{3}$ , $\lambda_\gamma > -\lambda_V$ saddle point otherwise
$F$	$0$	$1$	$0$	$\lambda_V = 0$	$-3, -3, -3$	stable node

TABLE I: Real and physically meaningful critical points of the autonomous system (28)-(30), their existence conditions, their corresponding eigenvalues and their stability conditions, in the case of exponential potential (26) and exponential coupling function (27). We have defined  $\alpha^\pm(\lambda_\gamma) \equiv \lambda_\gamma^2 \pm \lambda_\gamma \sqrt{\lambda_\gamma^2 - 6} > 3$ , as well as  $C^\pm \equiv 3 \left[ \pm \sqrt{2}\lambda_\gamma (\alpha^- - 3)^{-1/2} + \alpha^+ - 4 \right] / 4$  and  $D^\pm \equiv 3 \left[ \pm \sqrt{2}\lambda_\gamma (\alpha^+ - 3)^{-1/2} + \alpha^- - 4 \right] / 4$ .

Points	$w_\phi$	$q$	$c_s^2$	$Q_s$	$\epsilon$
$A$	1	2	1	3	3
$B$	1	2	1	3	3
$C$	$-1 + \frac{\alpha^+}{3}$	$-1 + \frac{\alpha^+}{2}$	0	arbitrary	$\frac{\alpha^+}{2}$
$D$	$-1 + \frac{\alpha^-}{3}$	$-1 + \frac{\alpha^-}{2}$	0	arbitrary	$\frac{\alpha^-}{2}$
$E$	$-1 + \frac{\lambda_V^2}{3}$	$-1 + \frac{\lambda_V^2}{2}$	1	$\frac{\lambda_V^2}{2}$	$\frac{\lambda_V^2}{2}$
$F$	-1	-1	1	0	0

TABLE II: Real and physically meaningful critical points of the autonomous system (28) - (30), and the corresponding values of the scalar field equation of state  $w_\phi$ , the deceleration parameter  $q$  and the instability-related parameters  $c_s^2$  and  $Q_s$ , which must be non-negative for a scenario free of ghosts and gradient instabilities, and the slow-roll parameter  $\epsilon$ .

$3 \times 3$  matrix  $\mathbf{Q}$  of the perturbation equations as we described above, and we extract its eigenvalues which are given in Table I too. Hence, we use them in order to deduce the stability properties. Furthermore, since we have the coordinates of the critical points of the autonomous system at hand, we can use them to calculate the corresponding  $w_\phi$  and  $q$  from (33), as well as  $Q_s$  and  $c_s^2$  from (34) and (35) respectively, and we present them in Table II. Finally, using the obvious relation  $\epsilon = q + 1$ , in the last column of Table II we present  $\epsilon$  of the corresponding critical points.

Let us now discuss the physical behavior of the above dynamical analysis. Since in this work we investigate inflation realization, first of all we are interested in those critical points where the expansion of the universe is accelerating, especially those with  $q \approx -1$ . Amongst them, we are interested in those points that are saddle or unstable, which means that if the universe starts from them, i.e. from inflation, the dynamics will naturally lead the universe away from them, viz. it will offer a natural exit from inflation [41, 45].

As we can see from Tables I and II, point  $E$  exhibits these features, and thus it corresponds to the inflationary solution we are looking for. Note that the physical quantities depend only on the potential exponent  $\lambda_V$ . In particular, the smaller  $\lambda_V$  is, the closer we are to de Sitter phase. Finally, note that both  $c_s^2$  and  $Q_s$  are positive there, which means that this inflationary solution is free of ghosts and potential instabilities. However, in this solution we obtain  $w_\phi = -1 + \lambda_V^2/3$ , which is always larger than  $-1$ , and thus correspondingly  $\epsilon$  is always positive definite along the stably inflationary trajectory (note that  $\epsilon \geq 0$  in all the obtained points). Therefore,  $n_T$  is not allowed to be positive



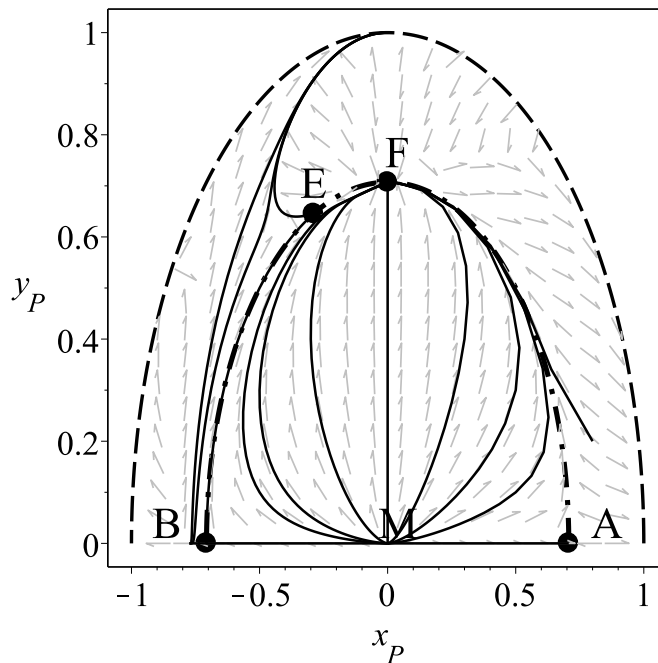


FIG. 1: Projection on the  $x_P$ - $y_P$  plane of the phase space behavior of the model (10), with  $V(\phi) = V_0 e^{\lambda_V \phi}$  and  $\gamma(\phi) = \gamma_0 e^{\lambda_\gamma \phi}$ , for  $p = 1$ ,  $\lambda_V = 1$  and  $\lambda_\gamma = -2$ . The region inside the inner semi-circle (seen as semi-ellipse in the figure scale), marked by the thick dashed-dotted line, is the physical part of the phase space. In this projection point  $E$  is saddle,  $F$  is an attractor,  $A$  and  $B$  are unstable, and the origin  $M$  is a saddle. The inflationary realization is described by point  $E$ .

in the model under consideration.

In order to see this behavior more transparently, we evolve the autonomous system (28) - (30) numerically for the choice  $\lambda_V = 1$  and  $\lambda_\gamma = 2$  and we present the resulting phase space behavior in Figure 1. For convenience, we project the phase space trajectories on the  $x_P$ - $y_P$  plane of the Poincaré variables  $x_P = x/\sqrt{1+x^2+y^2}$  and  $y_P = y/\sqrt{1+x^2+y^2}$ . As we can observe, the realization of inflation is described by the saddle point  $E$ , and the departure of the system from it after a finite time corresponds to the exit from inflation.

#### IV. CONCLUSIONS

In the present article we have studied in detail the theoretical challenge of single field inflation models to generate a blue tilt for the primordial gravitational waves. Considering a generalized single field inflation model with a Horndeski operator minimally coupled to Einstein gravity, we have performed a detailed phase space analysis and have shown explicitly that the only inflationary solution without any pathologies yields a positive definite value of the slow-roll parameter  $\epsilon$ . Therefore, up to leading order, the spectral index of the primordial tensor perturbations, which takes the form of  $-2\epsilon$  under the consistency relation, is always red tilted.

There might be directions to circumvent the theoretical difficulty pointed out in the present study, however this would require to extend into more complicated situations. For instance, one could try to go beyond the single slow-roll field. For instance, relation (7) can be altered by taking into account possible contributions that are higher-order in slow-roll [11], by including particle production effects [46], or by considering inflation models driven by some non-conventional matter such as in elastic inflation [47] and solid inflation [48]. However, the analyses of these possible complicated scenarios under the theoretical constraints imposed in Section II, need to be performed in detail in future projects, before accepting them as successful candidates for the description of Nature.

We end the present paper by clarifying our motivation. In this paper we pointed out a theoretically severe problem. That is, given a possible detection of a blue spectrum for primordial tensor fluctuations, single field inflation models under our current knowledge can hardly provide a reasonable interpretation. To demonstrate this theoretical difficulty, we have performed the dynamical system analysis in detail based on a class of G-inflation model of which the form is pretty generic. This issue might be circumvented by more complicated choices of the functions  $K$  and  $G$ , but

more problems associated with other instabilities/inconsistencies would appear, such as observational constraints of primordial non-gaussianities. We argue that, if the difficulty of realizing a blue tensor spectrum could eventually become a no-go theorem, then such a possible detection may spoil the picture of single field inflationary cosmology completely; otherwise, a more delicate model building is required. This is the goal of our following-up project.

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### Appendix A: Background dynamics

In this Appendix we provide the general forms of the equations of motion in the model under consideration. Varying the Lagrangian (8) with respect to the metric, one can obtain the Friedmann equations that determine the dynamics of the background universe as

$$H^2 = \frac{\rho_\phi}{3m_{\text{Pl}}^2}, \quad \dot{H} = -\frac{\rho_\phi + P_\phi}{2m_{\text{Pl}}^2}, \quad (\text{A1})$$

where in the general case the energy density and pressure write as

$$\rho_\phi = 2XK_X - K + 6G_XH\dot{\phi}X - 2XG_\phi, \quad (\text{A2})$$

$$P_\phi = K - 2X \left( G_\phi + G_X\ddot{\phi} \right), \quad (\text{A3})$$

respectively. In addition, varying (8) with respect to the scalar field yields the generalized Klein-Gordon equation

$$\begin{aligned} K_X \left( \ddot{\phi} + 3H\dot{\phi} \right) + 2K_{XX}X\ddot{\phi} + 2K_{\phi X}X - K_\phi - 2(G_\phi - G_{\phi X}X) \left( \ddot{\phi} + 3H\dot{\phi} \right) - 4G_{\phi X}X\ddot{\phi} \\ - 2G_{\phi\phi}X + 6G_X \left[ (HX)^\cdot + 3H^2X \right] + 6G_{XX}HX\dot{X} = 0, \end{aligned} \quad (\text{A4})$$

which is a second order differential equation and hence it is free of extra degrees of freedom. Now we can insert specific operators  $K$  and  $G$ , for example (10) and (11), into the above equations, and derive out straightforwardly the detailed equations of motion.

### Appendix B: Perturbation dynamics

In this Appendix we present the detailed expression of the quadratic action that characterizes the dynamics of the cosmological perturbations at linear order. For the curvature perturbation, the quadratic action is given by

$$S_2 = \int dt d^3x \frac{a}{2} z^2 \left[ \dot{\mathcal{R}}^2 - \frac{c_s^2}{a^2} (\nabla \mathcal{R})^2 \right], \quad (\text{B1})$$

where  $z^2$  and  $c_s^2$  are given by

$$z^2 = \frac{4a^2 m_{\text{Pl}}^4 \dot{\phi}^2}{\left(2m_{\text{Pl}}^2 H - \dot{\phi}^3 G_X\right)^2} \left[ K_X + \dot{\phi}^2 K_{XX} + 6H\dot{\phi}G_X + \frac{3\dot{\phi}^4 G_X^2}{2m_{\text{Pl}}^2} + 3H\dot{\phi}^3 G_{XX} - 2G_\phi - \dot{\phi}^2 G_{\phi X} \right], \quad (\text{B2})$$

$$c_s^2 = \frac{K_X + 4H\dot{\phi}G_X - \dot{\phi}^4 G_X^2 / (2m_{\text{Pl}}^2) - 2G_\phi + \dot{\phi}^2 G_{\phi X} + \left(2G_X + \dot{\phi}^2 G_{XX}\right) \ddot{\phi}}{K_X + \dot{\phi}^2 K_{XX} + 6H\dot{\phi}G_X + 3\dot{\phi}^4 G_X^2 / (2m_{\text{Pl}}^2) + 3H\dot{\phi}^3 G_{XX} - 2G_\phi - \dot{\phi}^2 G_{\phi X}}, \quad (\text{B3})$$

where the expression of  $z^2$  is related to Eq. (21) via  $z^2 = 2a^2 Q_s$ .

Under the specific  $K(\phi, X)$  and  $G(\phi, X)$  ansatzes of (10) and (11), as well as the slow-roll approximation, the above coefficients can be significantly simplified to leading order as

$$z^2 \approx 2a^2 m_{\text{Pl}}^2 \epsilon_\phi \frac{1 + 12p^2 \gamma (2\epsilon_\phi)^{p-1/2} \xi_H^p}{\left[1 - p\gamma (2\epsilon_\phi)^{p+1/2} \xi_H^p\right]^2}, \quad (\text{B4})$$

$$c_s^2 \approx \frac{1 + 8p\gamma (2\epsilon_\phi)^{p-1/2} \xi_H^p}{1 + 12p^2 \gamma (2\epsilon_\phi)^{p-1/2} \xi_H^p}. \quad (\text{B5})$$

We mention that  $z^2$  is required to be positively definite in order for the model to be free of any ghost mode. During inflation,  $c_s^2$  is also required to be positive and therefore the propagation of the primordial perturbations do not suffer from a gradient instability.

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